**United Kingdom
Mathematics Trust**

Junior Mathematical Olympiad
Tuesday 14 June 2022
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Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. Start each question on an official answer sheet on which there is a QR code.
5. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code.
6. Write on one side of the paper only. Make sure your writing and diagrams are clear and not too faint. (*Your work will be scanned for marking.*)
7. Arrange your answer sheets in question order before they are collected. If you are not submitting work for a particular problem, please remove the associated answer sheet.
8. Your answers should be fully simplified and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
9. Only answers are required to the questions in Section A.
10. For questions in Section B, you should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Junior Mathematical Olympiad should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

Section A

> Try to complete Section A within 30 minutes or so. Only answers are required.

A1. What is the value of $\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} + \frac{5}{6}$?

A2. Seven consecutive odd numbers add up to 105. What is the largest of these numbers?

A3. In a class, 55% of students scored at least 55% on a test. 65% of students scored at most 65% on the same test. What percentage of students scored between 55% and 65% (inclusive) on the test?

A4. What is the sum of the marked angles in this diagram?
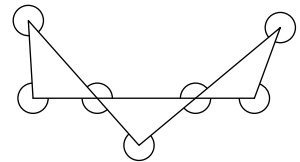
A5. Consider the six-digit multiples of three with at least one of each of the digits 0, 1 and 2, and no other digits. What is the difference between the largest and the smallest of these numbers?

A6. Two positive numbers a and b, with $a > b$, are such that twice their sum is equal to three times their difference. What is the ratio $a : b$?

A7. The diagram on the right shows a 4 by 4 square placed on top of a 5 by 5 square, so that they have one vertex in common as shown. One diagonal of each square is also drawn. What is the area of the shaded region that is inside the 4 by 4 square and between the two diagonals?
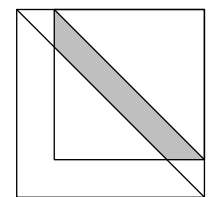
A8. The sum of the numbers 1 to 123 is 7626. One number is omitted so that the sum is now a multiple of 4. How many different numbers could be omitted?

A9. Dividing 52 by 12 gives 4 remainder 4. What is the sum of all the numbers for which dividing by 12 gives a whole number answer which is the same as the remainder?

A10. Farmer Alice has an alpaca, a cat, a dog, a gnu and a pig. She also has five fields in a row. She wants to put one animal in each field, but doesn't want to put two animals in adjacent fields if their names contain the same letter. In how many different ways can she place her animals?

Section B

> Your solutions to Section B will have a major effect on your result.
>
> Concentrate firstly on one or two Section B questions and then write out *full solutions* (not just brief 'answers'), including mathematical reasons as to why your method is correct.
>
> You will have done well if you hand in full solutions to two or more Section B questions.
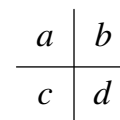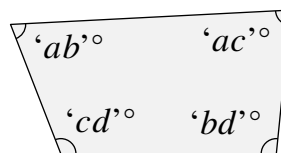>
> Do *not* hand in rough work.

B1. The sum of two numbers is 90.
40% of the first number is 15 more than 30% of the second number.

Find the two numbers.

B2. In a certain quadrilateral, the four angles are each two-digit numbers. These four numbers can be placed in the 2 by 2 grid shown, with one digit in each cell.

Find all the possibilities for the set of four angles.

B3. You start with a regular pentagon $ABCDE$. Then you draw two circles: one with centre A and radius AB, and the other with centre B and radius BA. Let the point inside the pentagon at which these two circles intersect be X.

What is the size of $\angle DEX$?

B4. Seth creates n standard dice by folding up n identical copies of the net shown. He then repeatedly puts one on top of another until there are none left, creating a vertical tower.
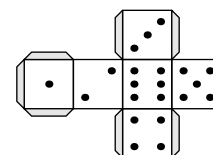
For each of the four vertical walls of the tower, he finds the total number of dots that are visible.

Given that the four totals calculated are all odd, what are the possible values for n?

B5. Charlie chooses one cell from a blank $n \times n$ square grid and shades it. The resulting grid has no lines of symmetry.

In terms of n, how many different cells could be shaded?

B6. The descriptors **'even'**, **'factors of 240'**, **'multiple of 3'**, **'odd'**, **'prime'** and **'square'** are to be placed in some order as row and column headings around the grid in positions a, b, c, d, e and f.

The digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 are to be placed in the empty cells inside the grid so that each digit satisfies both the relevant row and column headings.

(i) Show that it is possible to complete the grid.

(ii) In how many different ways can the grid be completed?